

PRENTICE'S RULE - ITS APPLICATIONS AND LIMITATIONS
(WITH SOME THOUGHTS ABOUT PLANO LENSES)

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One of the first things learned in the study of ophthalmic optics is Prentice's Rule; indeed, it is one of the most useful. That simple statement:

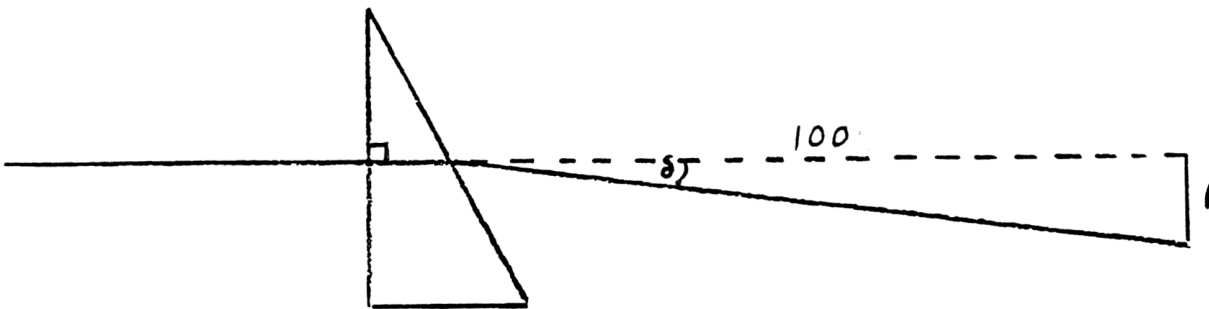
$$\text{Power X Decentration} \quad = \quad \text{Prism}$$

(in centimeters)

is surprisingly powerful. Through its use one can create a prismatic prescription through decentration using a finished uncut, thereby avoiding the time and expense of having to grind the Rx from a semi-finished blank. Or, one can adopt a processing system wherein all semi-finished blanks are blocked on geometric center, and then prism introduced so as to locate the optical center (or major reference point) at the desired location.

The latter application, especially, has received increasing attention in recent years as more prescription laboratories have adopted the "block-on-center" approach. The ready availability of computers has made this approach practical, since for compound lenses the mathematics involved in accurately calculating the required prism would otherwise be unwieldy. Even with computers, however, some laboratories have encountered difficulty with low powered prescriptions, where the amount of displacement of the optical center predicted through the use of Prentice's Rule doesn't work out in practice. This is with good reason, for the fact is that Prentice's Rule breaks down for ophthalmic lens powers of less than about ± 1.00 diopter.

In order to understand why this happens, let us look at how Prentice's Rule is derived. To do so we will start by examining the definition of a prism diopter. Simply stated, one prism diopter is a deviation of one in a hundred, under the condition where light passes through one surface perpendicularly.



1

Since we are dealing with a ratio, it doesn't matter what the units are. They can be meters, feet, or miles - as long as light deviates one part in one hundred.

We will use the symbols " δ " to indicate angle of deviation, and " Δ " to indicate prism power in prism diopters.

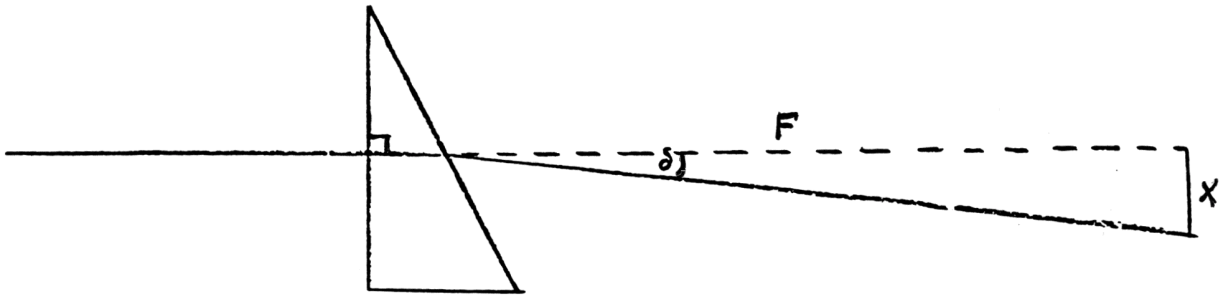


Fig 2

From Fig. 2 it can be seen that:

$$\tan \delta = \frac{x}{F} \quad (1)$$

Since one prism diopter is a deviation of 1 in 100, it follows that:

$$\Delta = 100 \tan \delta \quad (2)$$

Now let us consider a plano-convex lens positioned such that light enters perpendicularly to the plano side, and all refraction occurs at the curved surface:

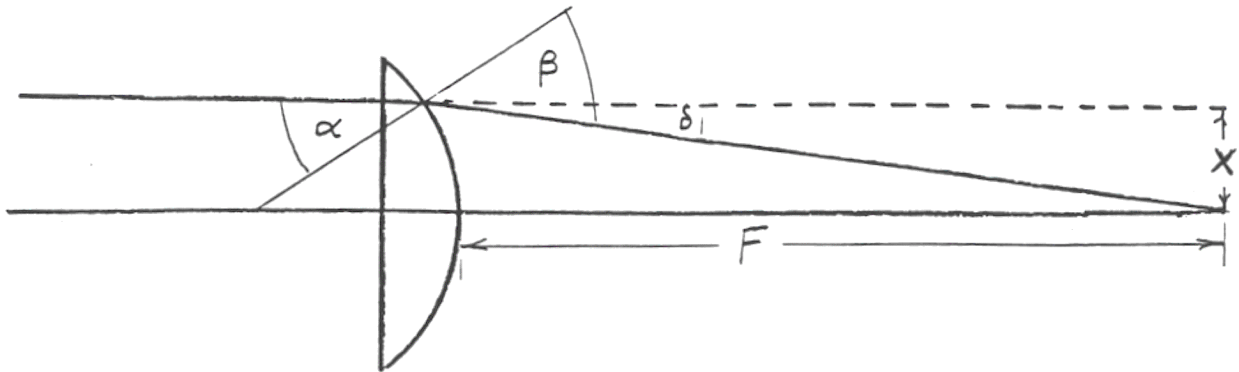


Fig. 3

Since "F" is now the back focal length of the lens, we know that:

$$D = \frac{1}{F} \quad (3)$$

where "D" is the back vertex power of the lens

From equations (1) and (2):

$$\Delta = 100 \tan \delta = (100) \left(\frac{X}{F} \right)$$

or

$$\Delta = (100) * (X) * (D) \quad (4)$$

This is Prentice's Rule:

“ Δ ” is the prism, in prism diopters, which equals

“ D ”, the back vertex power times

“ X ”, the decentration times

“100”, which converts from meters to centimeters.

We mentioned earlier that Prentice's Rule is not valid for low powered prescriptions; at least, for powers less than ± 1.00 diopter the results from applying Prentice's Rule can be misleading. Let's examine what that is.

Remember that our derivation of Prentice's Rule employed a lens where one surface was plano, with all the refraction taking place at the single curved surface. Indeed, the very definition of a prism diopter states that all refraction must occur at one surface. When measuring prism on a focimeter this condition is satisfied provided one surface is placed squarely against the focimeter stop (the use of a lens holding device may angle a prismatic lens such that the surface moves away from the stop, thereby invalidating both the prism and power readings). However, low powered ophthalmic lenses seldom employ a flat surface - the curves are likely to be in the vicinity of 6 diopters of surface power. This is why Prentice's Rule breaks down.

Consider the case of a plano ophthalmic lens having a convex curve of +6.00 diopters (1.53), a center thickness of 2.2mm, and fabricated of crown glass having an index of 1.523. If we perform the calculations, we find that the rear curve must be steeper than -6.00 diopters if the lens is to have zero power. Specifically:

Power	Plano
Center Thickness (mm)	2.2
Index	1.523
Front Curve (1.53)	+6.00
Rear Curve (1.53)	-6.052
Front Radius (mm)	88.333
Rear Radius (mm)	87.578

Note that the lens does not have the same curve on the front and rear surfaces. If it did, the lens would have plus power, since a curved front surface always gains plus power due to lens thickness.

We now begin to suspect that Prentice's Rule will not work for this plano lens, since that rule would state that no prism would be introduced by moving away from the optical center. For Prentice's Rule says:

$$\begin{array}{ccc} \text{Power} & \times & \text{Decentration} & = & \text{Prism} \\ \text{(Diopters)} & & \text{(Centimeters)} & & \text{(Prism Diopters)} \end{array}$$

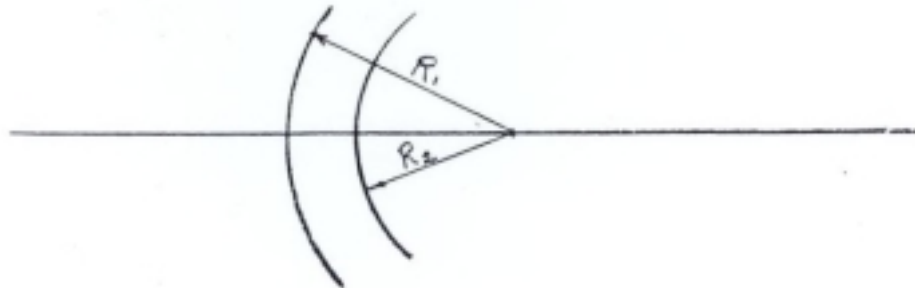
Since the power of a plano lens equals zero, Prentice's Rule predicts that prism should always equal zero for any amount of decentration. We suspect that is not the case.

But suppose we made our lens with the same amount of curvature on the front and rear surfaces. We now know that such a lens would have plus power. Using the same front curve, index, and center thickness as in our previous example, we find:

Power	+0.051
Center Thickness (mm)	2.2
Index	1.523
Front Curve (1.53)	+6.000
Rear Curve (1.53)	-6.000
Front Radius (mm)	88.333
Rear Radius (mm)	88.333

How about prism? If we were to place this lens on a focimeter, locate its optical center, and then move away from the optical center, we would find that prism would be introduced - but not the amount predicted by Prentice's Rule!

We now raise the questions "Is there a case where prism isn't introduced by moving the lens about against the focimeter stop?" The answer is "yes," and that case arises when the centers of curvature of the front and rear surfaces coincide - in other words, a concentric lens.



In this special case, it can be seen that the front and rear radii will differ and that difference will be exactly the center thickness of the lens. Again using the same front curve, center thickness, and index as in our previous examples:

Front Curve (1.53)	+6.00
Center Thickness (mm)	2.2
Index	1.523
Rear Curve (1.53)	-6.153
Power (diopters)	-0.100

Front Radius (mm)	88.333
Rear Radius (mm)	86.133

At this point we've learned some interesting things about plano lenses. Except for the special case where both surfaces are flat:

- **A plano lens does not have the same curvature on each surface.**
- **A plano lens does not have surfaces with concentric radii.**
- **A plano lens has a steeper concave curve than its convex curve.**
- **A plano lens does not conform to Prentice's Rule.**
- **A plano lens does have an optical center.**

If Prentice's Rule doesn't work for plano lenses, or for that matter for lenses near plano, how do we determine the amount of prism introduced by moving away from the optical center of such lenses? The answer is that we must involve the radii of the specific curves employed. Before discussing the equations used to perform such calculations, thus risking the loss of those readers who don't enjoy trigonometry, let's see how much prism is really introduced by moving 10mm away from the optical center of our plano and equi-sided examples.

	<u>Plano</u>	<u>Equi-Sided</u>
Front Curve (1.53)	+6.000	+6.000
Rear Curve (1.53)	-6.052	-6.000
Center Thickness (mm)	2.2	2.2
Index	1.523	1.523
Power (diopters)	0.000	+0.051
Prism 10mm from Optical Center (prism diopters)	0.100	0.148

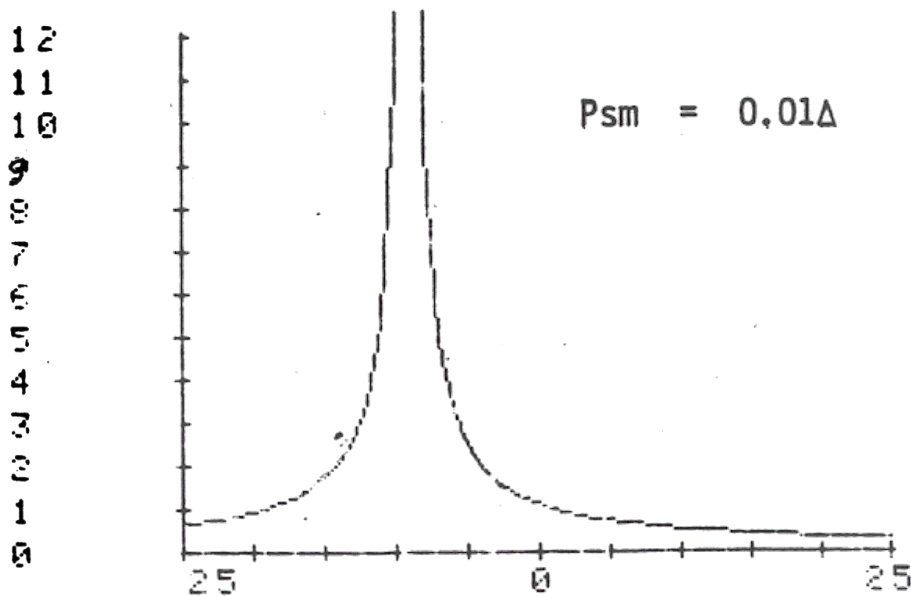
We didn't list our concentric example because we already know that no prism will be introduced by moving the lens about against the focimeter stop. This raises the question "Does a concentric lens have an optical center?" If we define "optical center" as the point (or points) of zero prism, then the answer is "yes" - it has an optical center, and that center exists everywhere on the lens. If any amount of prism is present, the lens cannot be concentric. Going further, if the difference between the radii of the front and rear surfaces equals the center thickness (as is the case with a concentric lens), but the centers of curvature do not exactly coincide, there will be prism everywhere on the lens - there will be no optical center.

So much for plano lenses - back to the application of Prentice's Rule. We now suspect that we should be wary of using it for low powers. Let's look at what really happens when we introduce 0.5 Δ of prism into ophthalmic lenses over a range of from +1.50 through -1.50 diopters, and compare it to what Prentice's Rule would predict. We will consider the lenses to be fabricated of ophthalmic crown glass (index = 1.523), plus lenses will have a -6.00 (1.53) concave curve and minus lenses will have a +6.00 (1.53) convex curve. We will use a 2.0mm edge thickness at a diameter of 60mm for plus lenses, and a 2.0mm center thickness for minus lenses.

<u>Power</u>	<u>Displacement of Optical Center as Predicted by Prentice's Rule</u>	<u>Actual Displacement of Optical Center</u>
+1.50 diopters	3.3 mm	3.0 mm
+1.25	4.0	3.6
+1.00	5.0	4.4
+0.75	6.7	5.7
+0.50	10.0	8.2
+0.25	20.0	14.4
Plano	∞	56.3
-0.25	20.0	32.1
-0.50	10.0	12.5
-0.75	6.7	7.8
-1.00	5.0	5.6
-1.25	4.0	4.4
-1.50	3.3	3.6

We now see that the departure from Prentice's Rule is quite substantial for the lower powers, and that the departure is greater for weak minus lenses than for weak plus lenses. This makes sense when we remember that a 6-base concentric lens has minus power. For a lens made of ophthalmic crown, at a center thickness of 2.0mm, it will have a power of -0.09 diopters. This might be thought of as the "critical value" for our 6-base glass lenses. Any amount of prism at all will throw the optical center off the lens entirely.

If we draw a graph where the "y" axis represents the amount of decentration resulting from the introduction of 0.01 diopters of prism, and the "x" axis represents lens power, it will look like this:



POWER (X) vs DECENTRATION (Y)

For a lens having a power of -0.25 diopters, one one-hundredth diopter of prism will move the optical center by more than one half millimeter. For a lens power of -0.125 diopters, this tiny amount of prism will move the optical center a full 4mm. At -0.09 diopters, a prism of 0.01 Δ will throw the optical center off the lens, regardless of its diameter!

Remember, the above graph, and the critical value of -0.09 diopters, apply only to 6-base glass lenses, 2.0mm thick. If the curves are steeper, the "critical point" will be more minus; if they are flatter, it will be less minus. Only if the base curve is flat, however, will it fall at zero as Prentice's Rule would predict.

So where does all this lead us? For one thing, it cautions us not to rely on Prentice's Rule if we are setting up a surfacing system wherein we plan to block on geometric center, and position the optical center by introducing prism at blocking or generating. For another, it tells us not to be too concerned if the optical center of a -0.25 diopter lens (or a lens in which a meridional power is -0.25 diopters) isn't where we expect it to be.

For those who have an aversion to trigonometry, this might be a good place to stop reading. For those who are interested in learning how to calculate the amount of prism required to move the optical center a given amount, the equations are as follows:

- Given:
- D_1 = Front Surface Power (1.53)
 - D_2 = Rear Surface Power (1.53)
 - t = Center Thickness (mm)
 - n = Index of Refraction

$$x = \text{Desired Decentration (mm)}$$

$$\text{To find: } \Delta = \text{Required prism (prism diopters)}$$

$$R_1 = \frac{530}{D_1}$$

$$R_2 = \frac{530}{D_2}$$

$$w = \sin^{-1} \left(\frac{x}{R_2} \right)$$

$$z = R_2 - R_1 + t$$

$$\alpha = \sin^{-1} \left(\frac{z \sin w}{R_1} \right)$$

$$\beta = \sin^{-1} (n \sin \alpha)$$

$$\delta = \beta - \alpha$$

$$\Delta = 100 \tan \delta$$

Conversely, if we want to determine how much decentration will result from a given amount of prism:

$$\text{Given: } D_1 = \text{Front Surface Power (1.53)}$$

$$D_2 = \text{Rear Surface Power (1.53)}$$

$$t = \text{Center Thickness (mm)}$$

$$n = \text{Index of Refraction}$$

$$\Delta = \text{Prism (prism diopters)}$$

$$\text{To find: } x = \text{Resulting Decentration (mm)}$$

$$\delta = \tan^{-1} \left(\frac{\Delta}{100} \right)$$

$$\alpha = \tan^{-1} \frac{(\sin \delta)}{n - \cos \delta}$$

$$z = R_2 - R_1 + t$$

$$w = \sin^{-1} \frac{(R_1 \sin \alpha)}{z}$$

$$x = R_2 \sin w$$

On reviewing this paper, one of my colleagues pointed out that it is possible to derive a generalized version of Prentice's Rule which, while slightly more involved than the original version, avoids trigonometric expressions and works well for low powers. Indeed, for the weaker powers where Prentice's Rule breaks down, the involved angles become small enough so that we can let the sine of the angle equal the angle itself. By so doing, we derive the following equations:

$$\Delta = 100 (P_1 + P_2 - \frac{t}{(n-1)} P_1 + P_2) x$$

or

$$x = \frac{\Delta}{100 (P_1 + P_2 - \frac{t}{(n-1)} P_1 + P_2)}$$

Where:

$$P_1 = \text{True Front Surface Power} = \frac{1000 (n-1)}{R_1}$$

$$P_2 = \text{True Rear Surface Power} = \frac{1000 (n-1)}{R_2}$$

$$t = \text{Center Thickness (meters)}$$

$$x = \text{Decentration (meters)}$$

$$\Delta = \text{Prism (prism diopters)}$$

Note: P_1 and P_2 are true surface powers (not 1.53 powers). In the usual situation where the rear surface is concave, P_2 should be considered a minus quantity in these equations.

For all low powered ophthalmic lenses, and even for most higher powers, the above equations provide more accuracy than can be achieved with the usual version of Prentice's Rule. For weak lenses, these equations are sufficiently accurate for most any practical ophthalmic application.